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Gluon distribution and $dF_2/d\ \ln Q^2$ at small x in the next-to-leading order

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Abstract. We obtain an approximate form of gluon momentum density $G(x, Q^2)$ from next to leading order (NLO) GLAP equation at low x, with a factorization ansatz recently reported and test its validity by comparing it with that of Glück, Reya and Vogt (GRV-HO) which has no such additional assumption. Using Prytz's approximate method, we calculate $dF_2/d\log Q^2$ using LO and NLO forms of gluon density. Limitation of the present formalism is critically discussed and approximate factorizability zone in $(x - Q^2)$ plane is indicated.

1 Introduction

Traditionally, information on the gluon shape is extracted directly from the large p_T prompt photon production in fixed target p_N collisions, where $qg \to \gamma q$ process ($q =$ quark, $g =$ gluon) is dominant [1]. In recent years however, the measurement of proton structure function at $e-p$ collider HERA in the regime of low x ($x \leq 10^{-4}$) has opened a new direction in this problem [2, 3]. At low x, structure function $F_2(x, Q^2)$ is dominated by gluons and the GLAP equations [4–7] can be approximately solved [8–10] so that the Q^2 derivative of $F_2(x, Q^2)$ can be directly related to the gluon momentum density $G(x, Q^2)$. Similarly, measurement of the longitudinal structure function $F_L(x, Q^2)$ has also long been advocated [11, 12] as a direct probe of the gluon density at small x.

The structure of the gluon momentum density and its x and Q^2 evolutions have been reported in recent years both in GLAP [13, 14] and BFKL approaches [15–18]. The aim of the present paper is to suggest an alternative form of gluon evolution using the next-to-leading order (NLO) GLAP equation [7, 19–20] with an additional assumption of factorization [21, 22] and test its validity by comparing it with that of Glück, Reya and Vogt (GRV-HO) [23] which does not have such factorizability in x and t $\left(t = \log \frac{Q^2}{\Lambda^2}\right)$. We then study its consequences by estimating the slope of the structure functions at low x [8, 9]. Section 2 outlines the formalism while Sect. 3 is devoted to results and discussions.

2 Formalism

(A) Gluon momentum density

The gluon evolution equation with the next-to-leading order (NLO) effects is [7, 19, 20]

$$Q^2 \frac{\partial G(x, Q^2)}{\partial Q^2} = \int_x^1 \frac{\alpha(Q^2)}{2\pi} \left[P_{gg}^{(1)}(z) + \frac{\alpha(Q^2)}{2\pi} P_{gg}^{(2)}(z) \right] G(x/z, Q^2) dz \tag{1}$$

where the splitting kernels $P_{gg}^{(1)}(z)$ of LO and $P_{gg}^{(2)}(z)$ of NLO are defined in [7, 19, 20] while the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the form in NLO as

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \left[1 - \frac{\beta_1 \ln \ln \frac{Q^2}{\Lambda^2}}{\beta_0^2 \ln \frac{Q^2}{\Lambda^2}} \right] \tag{2}$$

with $\beta_0 = \frac{1}{3}(33 - 2n_f)$ and $\beta_1 = 102 - \frac{38}{3} n_f$, n_f being the number of flavour. In (1), $G(x, Q^2) = xg(x, Q^2)$ is the gluon momentum density and $g(x, Q^2)$ is the gluon number density of proton.

Let us use the variable $\bar{t}$ [20]

$$\bar{t} = \frac{2}{\beta_0} \ln \left[\frac{\alpha(Q_0^2)}{\alpha(Q^2)} \right] \tag{3}$$

$$\text{so that } \frac{\alpha(\bar{t})}{2\pi} = \frac{\alpha(\bar{t}_0)}{2\pi} \exp\left[-\frac{\beta_0}{2} \bar{t} \right] . \tag{4}$$

Using (3) and (4) in (1), we get

$$\frac{dG(x, \bar{t})}{d\bar{t}} = \int_0^1$$

$$\left[P_{gg}^{(1)}(z) + \frac{\alpha(\bar{t})}{2\pi}\left\{P_{gg}^{(2)}(z) - \frac{\beta_1}{2\beta_0}P_{gg}^{(1)}(z)\right\}\right] G(x/z,\bar{t})dz\,. \tag{5}$$

Let us now assume that the x and t dependence of the structure function is factorizable [21, 22].

$$G(x,\bar{t}) = U(x)\ h(\bar{t}) \tag{6}$$

with the condition

$$U(x) = (G(x,t_0)) \tag{7}$$

so that (5) becomes

$$U(x)\frac{d\,\bar{h}(t)}{d\,\bar{t}} = \int_x^1 \left[P_{gg}^{(1)}(z) + \frac{\alpha(\bar{t}_0)}{2\pi}e^{-\frac{\beta_0}{2}\bar{t}} \left\{P_{gg}^{(2)}(z) - \frac{\beta_1}{2\beta_0}P_{gg}^{(1)}(z)\right\}\right] U(x/z)\ h(\bar{t})dz \tag{8}$$

$$\text{or}\quad \frac{d\,h(\bar{t})}{h(\bar{t})} = \left[\int_x^1 P_{gg}^{(1)}(z)\frac{U(x/z)}{U(x)}dz + \frac{\alpha(t_0)}{2\pi} \int_x^1 \left\{P_{gg}^{(2)}(z) - \frac{\beta_1}{2\beta_0}P_{gg}^{(1)}(z)\right\} e^{-\frac{\beta_0\bar{t}}{2}}\frac{U(x/z)}{U(x)}dz\right] d\bar{t} \tag{9}$$

$$\text{Let } I_1(x) = \int_x^1 P_{gg}^{(1)}(z)\frac{U(x/z)}{U(x)}dz \tag{10}$$

$$\text{and } I_2(x) = \int_x^1 \left[P_{gg}^{(2)}(z) - \frac{\beta_1}{2\beta_0}P_{gg}^{(1)}(z)\right]\frac{U(x/z)}{U(x)}dz\,. \tag{11}$$

Using (10) and (11) in (9), we get

$$\frac{d\,h(\bar{t})}{h(\bar{t})} = \left[I_1(x) + I_2(x)\frac{\alpha(\bar{t}_0)}{2\pi}e^{-\frac{\beta_0}{2}\bar{t}}\right] d\bar{t}\,, \tag{12}$$

the solution of which is

$$h(\bar{t}) = h(\bar{t}_0)\ \exp\left[I_1(x)\bar{t} - \frac{2}{\beta_0}I_2(x)\frac{\alpha(\bar{t})}{2\pi}\left(1 - e^{+\beta_0\frac{t}{2}}\right)\right]. \tag{13}$$

This yields the gluon momentum density with second order effects as

$$G^{(2)}(x,\bar{t}) = G(x,\bar{t}_0)\left[\frac{\alpha(\bar{t}_0)}{\alpha(\bar{t})}\right]^{I_1(x)^2/\beta_0} \exp\left[I_2(x)\frac{2}{\beta_0}\cdot\frac{\alpha(\bar{t})}{2\pi}\left\{\frac{\alpha(\bar{t}_0)}{\alpha(\bar{t})} - 1\right\}\right]. \tag{14}$$

We note that the leading term of the splitting kernel $P_{gg}^{(1)}(z)$ as $z \to 0$ is

$$P_{gg}^{(1)}(z) \sim \frac{6}{z} \tag{15}$$

$$\text{so that } I_1(x) \sim 6\ \ln\frac{1}{x} \tag{16}$$

As the running coupling constant $\frac{\alpha_s(Q^2)}{2\pi}$ retains only the first term of (2) one identifies the leading order term of the gluon momentum density as

$$G^{(1)}(x,t) = G(x,t_0)\left[\frac{\alpha^{LO}(Q_0^2)}{\alpha^{LO}(Q^2)}\right]^{I_1(x)\,2/\beta_0} \tag{17}$$

which has $x \to 0$ limit

$$G^{(1)}(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^{\frac{12}{\beta_0}\ \ln\ \frac{1}{x}} \tag{18}$$

where $t = \ln\frac{Q^2}{\Lambda^2}$.

Taking the leading term of the splitting kernel $P_{gg}^{(2)}(z)$ as [9]

$$P_{gg}^{(2)}(z) \sim \frac{52}{3}\ \frac{1}{z}\,, \tag{19}$$

$x \to 0$ limit of $I_2(x)$ becomes

$$I_2(x) \sim \left(\frac{52}{3} - \frac{3\beta_1}{\beta_0}\right)\ \ln\frac{1}{x}\,. \tag{20}$$

This yields finally at low x

$$G^{(2)}(x,t)\ = G(x,t_0)\left[\frac{\alpha(t_0)}{\alpha(t)}\right]^{\frac{12}{\beta_0}\ \ln\ \frac{1}{x}} \exp\ \left[\left(\frac{52}{3} - \frac{3\beta_1}{\beta_0}\right)\ \ln\frac{1}{x}\cdot\frac{2}{\beta_0}\ \frac{\alpha(t)}{2\pi}\ \left\{\frac{\alpha(t_0)}{\alpha(t)} - 1\right\}\right]. \tag{21}$$

Equations (14), (17), (18) and (21) are our main results.

Let us now discuss how the present results differ from the standard results of the gluon evolutions at low x.

The factorization ansatz, (6) is too strong an assumption and cannot be proved in general within GLAP [4–7] or BFKL [15–18] dynamics. If only the $1/z$ part of the $P_{gg}(z)$ is retained, one obtains such factorizability for log $G(x,t)$. In NLO, even log $G(x,t)$ loses such factorizability with such double leading log term. The assumption (6) which leads to (21) thus warrants phenomenological testing before making predictions from it. In the following, we will, therefore extend our earlier leading order analysis [22] and attempt to see how the predictions with this assumption compare with those of gluon distribution which does not have such an additional assumption like the HO model of Glück, Reya and Vogt [GRV-HO] [23]. This will enable us to find the kinematic region of approximate validity of (21).

(B) Structure function at low x from gluon density

Sometime back [8], a method to obtain an approximate relation between the gluon density and the scaling violation of $F_2(x,Q^2)$ at low x has been reported, leading to a formula,

$$\frac{d\ F_2(x)}{d\ \log Q^2} \approx \frac{\alpha_s}{4\pi}\cdot\frac{20}{9}\cdot G(2x) \tag{22}$$

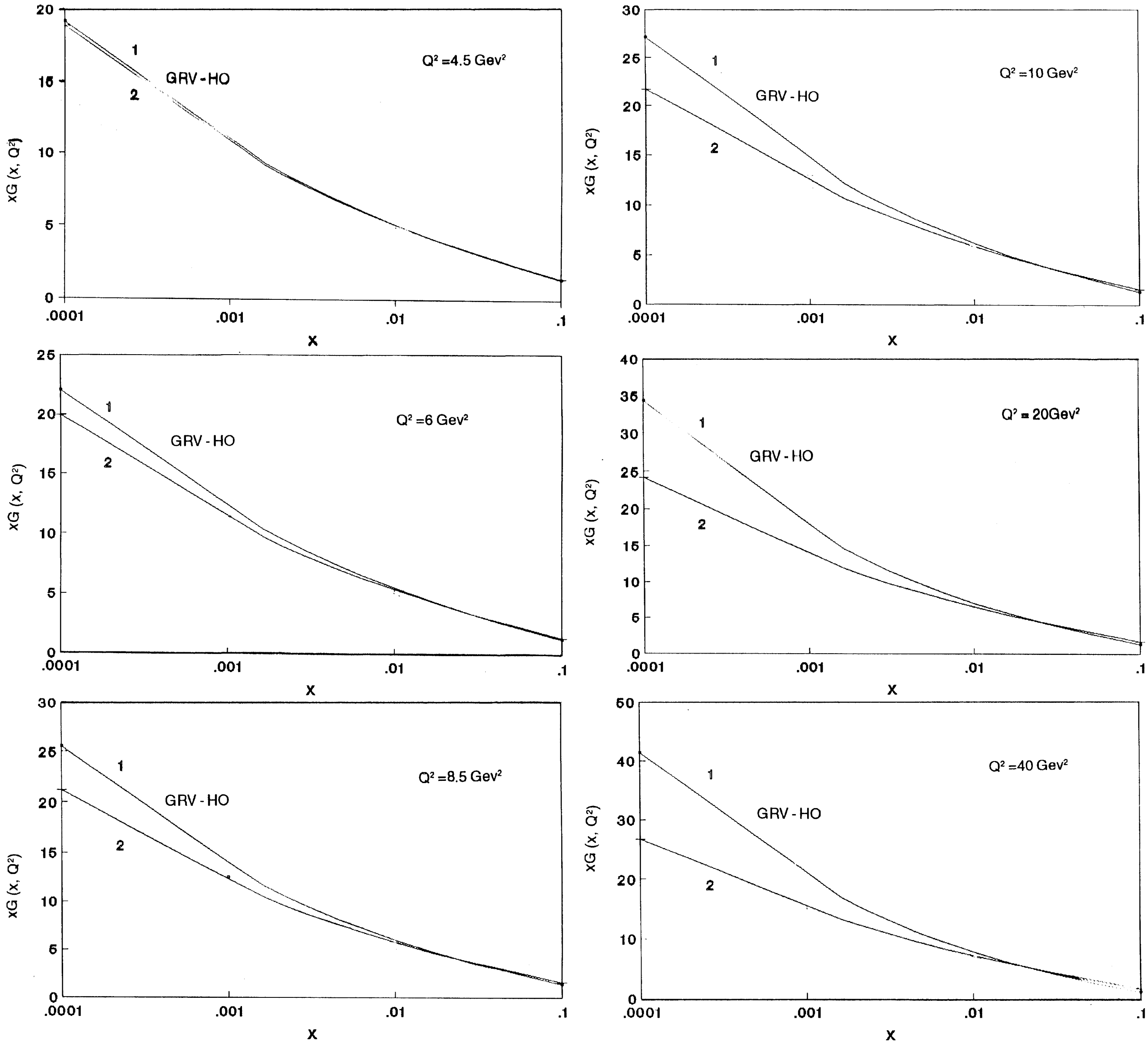

Fig. 1a–l. $G(x, Q^2)$ as function of x at Q^2 (in GeV^2) values **a** 4.5; **b** 6; **c** 8.5; **d** 10; **e** 20; **f** 40; **g** 80; **h** 100; **i** 160; **j** 1600; **k** 10^4; **l** 10^5. Curves 1 and 2 represent $G(x, Q^2)$ obtained form GRV-HO [23] and (14)

The method has later been extended [9] to include the NLO corrections as well:

$$\frac{d\ F_2(x)}{d\ \log Q^2} \approx G(2x) \cdot \frac{20}{9} \cdot \frac{\alpha_s}{4\pi}\left[\frac{2}{3} + \frac{\alpha_s}{4\pi} \cdot 3.58\right] + \left(\frac{\alpha_s}{4\pi}\right)^2 \cdot \frac{20}{9} \cdot N(x, Q^2)\ , \tag{23}$$

where $N(x, Q^2)$ is given explicitly in (7) of [9].

Equations (22) and (23) have been used to measure the gluon momentum density at low x. In the present work, we rather follow an inverse approach: we estimate the logarithmic slope of the structure function from the proposed gluon evolutions as discussed earlier.

3 Results and discussions

As noted earlier, the factorization assumption (6) is in general not true in GLAP [4–7] or even in BFKL [15–18] dynamics. We have therefore, attempted to see how the predictions with this assumption compare with those of gluon distribution which does not have such an assumption as in GRV-HO [23]. This will enable us to find the kinematic region of its approximate validity.

In Fig. 1 (a–l), we show the prediction of (21) with factorization ansatz (6) [Curve marked 2] and compare with GRV-HO [23] [Curve marked 1] for representative Q^2 values 4.5, 6, 8.5, 10, 20, 40, 80, 100, 160, 1600, 10^4 and $10^5\,\mathrm{GeV}^2$ and $10^{-4} < x < 10^{-1}$ starting with the evolution at $Q_0^2 = 4\,\mathrm{GeV}^2$. These figures show the following

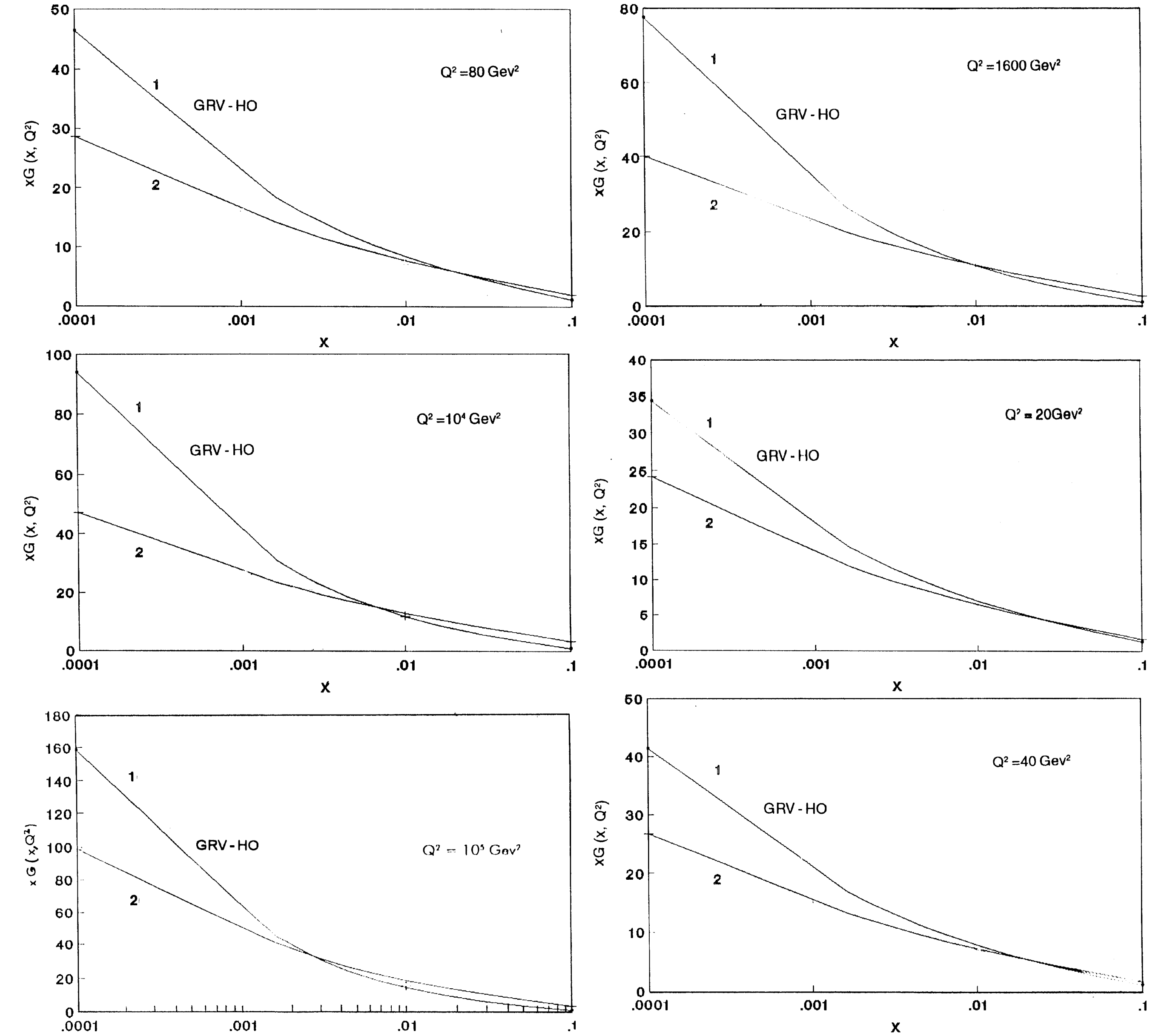

Fig. 1a–l. (continued)

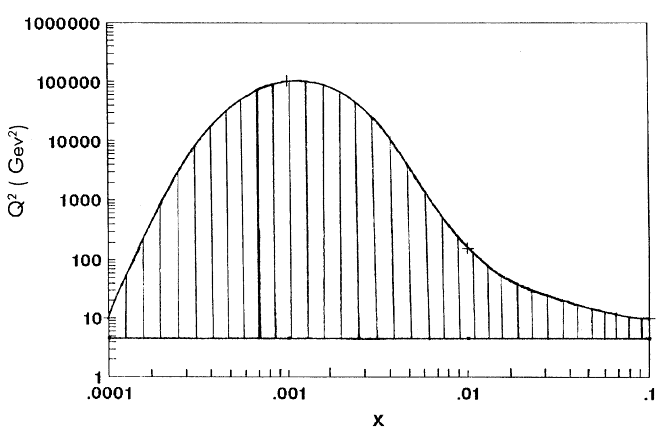

Fig. 2. Approximate factorizability zone in x-Q^2 plane

feature for smaller x range ($x < 10^{-2}$): at fixed Q^2, the difference between the two increases as x is decreased, such difference becomes more as Q^2 is increased. For each Q^2, there is a cross-over point for both the curves where both the predictions are numerically equal. The cross-over point shifts to lower x as Q^2 increases. Approximately, such cross over occurs at $x > 10^{-2}$ for $Q^2 \sim 4.5$–$160\,\mathrm{GeV}^2$ and at $10^{-3} < x < 10^{-2}$ for $Q^2 \sim 1600$–$10^5\,\mathrm{GeV}^2$. This feature was observed earlier for the corresponding LO analysis as well [22]. From an analysis of these figures, we can find the limited range of x and Q^2 where our approximate form of factorizable gluon density differs from nonfactorizable GRV-HO [23] by not more than 20% as shown in Fig. 2.

The shape of the gluon distribution in LO can be tested through its approximate relationship with the longitudinal structure function F_L [11]. However, as there is yet

Table 1. Predicted values of the logarithmic slope $d\ F_2/d\ \ln Q^2$ and their comparison with data [29] at $Q^2 = 20\,\mathrm{GeV}^2$

x	Measured value of slope	Predicted values of the slope		
		GRV	MRS Do'	MRS D'$_-$
		LO	LO + NLO	LO + NLO
8.5×10^{-4}	0.45 ± 0.08	0.496	0.49	0.58
2.55×10^{-3}	0.30 ± 0.09	0.307	0.41	0.44
2.68×10^{-3}	0.25 ± 0.05	0.300	0.32	0.33
4.65×10^{-3}	0.23 ± 0.04	0.230	0.25	0.25

Table 2. Percentage of difference between theory and experiment for logarithmic slope $d\ F_2/d\ \ln Q^2$ from Table 1

x	GRV (LO)	MRS Do'	MRS D'$_-$
8.5×10^{-4}	10.2%	8.8%	28.8%
2.55×10^{-3}	2.3%	36.6%	46.6%
2.68×10^{-3}	20%	28%	32%
4.65×10^{-3}	0%	8.6%	8.6%

no such simple relationship in NLO [24–26], we rather appeal to the approximate relationships (22) and (23) between the logarithmic slope of the structure function and the gluon distribution. For (22) we use (17) and GRV-LO input [27] while for (23), we use (14) taking MRS Do$'$ and D$'_-$ parametrizations [28] which incorporate NLO effects. For both cases, we take $\Lambda = 263\,\mathrm{Mev}$ [29].

The results are shown in Table 1 and compared with data [29] at $Q^2 = 20\,\mathrm{GeV}^2$. The expected trend of increase of $dF_2/d\ \log Q^2$ as x decreases is clearly manifested in Table 1. However, both the LO and the NLO results are higher than the experiment. The percentage of this difference is given in Table 2, which shows that the difference is enhanced for NLO results.

Let us discuss quantitatively the results of Table 2. At $20\,\mathrm{GeV}^2$ the cross-over point of GRV-HO [23] and (14) occurs at $x > 10^{-2}$ while the data [29] are all at $x < 10^{-2}$ where the difference between the two is considerable [Fig. 1(e)], implying significant contribution from non-factorizable component of the gluon distribution. For LO results too [22], the cross over is roughly at the same point while the difference between the predictions of GRV-LO and the present one (17) is slightly less. Ideally it would have been more appropriate to compare the prediction of slope at $x > 10^{-2}$ near the cross-over point where the non-factorizable component is less dominant. However, lack of data makes this possibility rather academic.

To conclude, we have proposed a form of gluon distribution at low x which is based on factorization of its x and t dependences. Introduced initially as a convenient way [21] of obtaining approximate analytical solutions of GLAP equations, we have tested its validity by comparing with a gluon distribution which has no such factorizability. For computational simplicity we have chosen GRV [27, 23] in LO and NLO versions. For each Q^2, we have obtained a cross-over point for the two distributions. Around the cross-over points, non-factorizable component tends to be negligible. On this basis we have obtained an approximate factorizable zone [Fig. 2]. Our analysis suggest that non-factorizable component invariably tends to enhance (deplete) the gluon distribution for x below (above) the cross-over points. lt will be interesting to investigate dynamical basis of this feature of gluon distribution.

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References

1. A.D. Martin, W.J. Stirling, R.G. Roberts, Phys. Rev. **D50** (1994) 6734
2. T. Ahmed et al., HI Collaborations, Nucl. Phys. **B439** (1995) 471
3. M. Derrick et al., ZEUS Collaboration, Z. Phys. **C65** (1995) 379
4. V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 438
5. L.N. Lipatov, Sov. J. Nucl. Phys. **20** (1975) 94
6. Yu.L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641
7. G. Altarelli, G. Parisi, Nucl. Phys. **B126** (1977) 298
8. K. Prytz, Phys. Lett. **B311** (1993) 286
9. K. Prytz, Phys. Lett. **B332** (1994) 393
10. K. Bora, D.K. Choudhury, Phys. Lett. **B354** (1995) 151
11. A.M. Cooper Sarkar et al., Z Phys. **C39** (1988) 281
12. A.V. Kotikov, G. Parente, preprint (hep-ph/9606409)
13. A. De'Rujula, S.L. Glashow, H.D. Politzer, S.B. Trieman, F. Wilczek and A. Zee, Phys. Rev. **D10** (1974) 1649
14. R.D. Ball, S. Forte, Phys. Lett. **B335** (1994) 77; Phys. Lett. **B336** (1994) 77
15. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Zh. Eksp. Teor. Fiz. **72** (1977) 373; Sov. Phys. JETP **45** (1977) 199
16. Ya.Ya. Balitskij, L.N. Lipatov, Yad, Fiz. **28** (1978) 1597; Sov. J. Nucl. Phys. **28** (1978) 822
17. F.C. Coriano, A.R. White, Phys. Rev. Lett. **74** (1995) 4980
18. For phenomenological analysis with BFKL equations see A.J. Askew, J. Kwiecinski, A.D. Martin, P.J. Sutton, Phys. Rev. **D47** (1993) 3775; Mod. Phys. Lett. **A8** (1993) 3813; Phys. Rev. **D49** (1994) 4402
19. W. Furmanski, R. Petronzio, Phys. Lett. **B97** (1980) 437
20. W. Furmanski, R. Petronzio, Z. Phys. **C11** (1982) 293
21. A. Saikia, D.K. Choudhury, Pramana J. Phys. **38** (1992) 313
22. R. Deka, D.K. Choudhury, Z. Phys. **C75**, 679–683 (1997), hep-ph/9707247
23. M. Glück, E. Reya, A. Vogt, Z. Phys. **C67** (1995) 438
24. E.B. Zijlstra, W.L. van Neerven, Nucl. Phys. **B383** (1992) 552
25. E.L. Berger, R. Meng. Phys. Lett. **B304** (1993) 318
26. J.Blümlein, Nucl. Phys. B., Proc. Suppl. **39 BC** (1995) 22
27. M. Glück, E. Reya, A. Vogt, Z. Phys. **C53** (1992) 127
28. A.D. Martin, W.J. Stirling, R.G. Roberts, Phys. Lett. **B306** (1993) 145
29. ZEUS Collaboration, DESY 94–192